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Finite size giant magnons in the $SU(2) \times SU(2)$ sector of $AdS_4 \times \mathbb{CP}^3$

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Abstract

We use the algebraic curve and Lüscher's μ-term to calculate the leading order finite size corrections to the dispersion relation of giant magnons in the $SU(2) \times SU(2)$ sector of $AdS_4 \times \mathbb{CP}^3$. We consider a single magnon as well as one magnon in each SU(2). In addition the algebraic curve computation is generalized to give the leading order correction for an arbitrary multi-magnon state in the $SU(2) \times SU(2)$ sector.

Contents

1 Introduction

During the last decade, a large amount of work has been put into the understanding of the duality between $\mathcal{N} = 4$ super Yang-Mills and type IIB string theory on $\mathrm{AdS}_5 \times S^5$ [1, 2, 3]. An important discovery was that the theories on both sides of this correspondence are governed by integrable structures [4, 5, 6, 7, 8, 9].

Motivated by the development of new superconformal world-volume theories for multiple M2-branes [10, 11, 12, 13], Aharony, Bergman, Jafferis, and Maldacena recently proposed a new class of superconformal field theories in 2+1 dimensions with $\mathcal{N} = 6$ supersymmetry, which are conjectured to describe N interacting M2-branes in a background of $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ [14, 15]. These ABJM theories have $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory, with Chern-Simons terms at level k for the gauge fields, and allows a 't Hooft limit where $k, N \to \infty$ with the coupling $\lambda = N/k$ fixed. In the large k limit, the membrane theory is compactified so that the dual theory is given by type IIA string theory on an $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background.

Part of the success in the studies of the $\mathrm{AdS}_5/\mathrm{SYM}_4$ duality lies in the identification of the fundamental excitations in the two theories. In the weak coupling regime these are magnons propagating along the gauge theory spin-chain [4]. At large coupling, magnons with finite momentum evolve into giant magnons [16], describing localized solitonic excitations on the world-sheet. The integrability of the theories was essential in these calculations.

Remarkably, integrable structures seem to appear also in the new AdS_4/CFT_3. Minahan and Zarembo [17] showed that the two-loop dilation operator of the scalar SU(4) sector of the Chern-Simons theory is equivalent to an integrable Hamiltonian, and conjectured a set of Bethe equations valid for the full two-loop theory (see also [18]). At strong coupling, the type IIA action has been formulated in terms of a super-coset sigma model [19, 20], and using the pure spinor formalism [21, 22]. Additionally an algebraic curve has been constructed [23]. Both of these limits are incorporated in the proposed all-loop generalization of the Bethe equations [24]. These Bethe equations have also been derived from the proposed exact S-matrix of the theory [25].[1]

The spin-chain of ABJM differs from that of $\mathcal{N} = 4$ SYM in that the SU(4) representations alternate between adjacent sites.[2] The spin-chain ground state preserves an $SU(2|2)$ subgroup of the full $OSp(2,2|6)$ symmetry of the gauge theory. The fundamental excitations fall into two $(2|2)$ multiplets [25, 31]. In addition there are quasi-bound states. The theory has an important closed $SU(2)\times SU(2)$ subsector, which includes one excitation from each fundamental multiplet.

At strong coupling, the spin-chain ground state corresponds to a point-like string spinning on a great circle of each sphere [23, 31, 32, 33]. World-sheet excitations above this ground state have been studied in the plane wave limit [31, 32, 34]. Additionally, two different kinds of giant magnons have been found. The first one is in $\mathbb{R} \times S^2 \times S^2$, where the magnons live on one or both of the spheres [31, 34, 35, 36, 37, 38]. The other giant magnon solution is spinning on $\mathbb{R} \times \mathbb{RP}^2$ [31, 36]. In this paper, only the first kind of magnons will be considered.

In recent years, one aspect of the AdS_5/SYM_4 duality that has attracted much interest is that of finite size corrections and wrapping interactions. The gauge theory spectrum derived from the Bethe equations is valid only for asymptotically large operators. For finite size operators, corrections are expected to arise [39]. Recently the four loop corrections stemming from wrapping interactions have been calculated directly from the gauge theory [40, 41, 42], as well as using the thermodynamic Bethe ansatz (TBA) and the Lüscher formulae [43].

On the string theory side, finite size corrections to the giant magnon dispersion relation have been studied using direct sigma model calculations [44, 45], Lüscher formulae [46, 47], the algebraic curves [48] and analogies with the sine-Gordon equation [49].

For the $AdS_4\times\mathbb{CP}^3$ theory, finite size effects in the Penrose limit have been considered [50], and the finite size corrections to the giant magnon dispersion relation have been calculated for the case of two $SU(2)\times SU(2)$ magnons with equal momenta [35, 51, 52]. In this paper we will consider finite size corrections to more general multi-magnon states in the $SU(2)\times SU(2)$ sector. The calculation of finite size effects using different formulations of the theory pose a good consistency check.

While this paper was being prepared, we received [53] which contains results that overlap with parts of this paper.

[1] Recently a mismatch between the string theory and Bethe ansatz results for the one-loop correction to spinning strings. See [26, 27, 28, 29, 30] for discussions of this issue.

[2] Another important difference is that the scalars in ABJM transform as a $\mathbf{4}$ or a $\bar{\mathbf{4}}$ under the SU(4) R-symmetry, while in $\mathcal{N} = 4$ SYM they transform as a $\mathbf{6}$.

2 Finite size corrections from the algebraic curve

The algebraic curve for giant magnons in $\mathrm{AdS}_5 \times S^5$ was first given in [54], and was discussed in more detail in [55]. In [48], the curve for a finite size magnon was constructed. Finite size corrections were also discussed in a finite gap context in [56, 57]. In this section we build upon these solutions to obtain the energy shift for finite size giant magnons in the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ Chern-Simons theory.

2.1 The algebraic curve

Using the algebraic curve of [23], a classical string state in $\mathrm{AdS}_4 \times \mathbb{CP}^3$ is mapped to a ten-sheeted Riemann surface. The branches $q_i(x)$, $i = 1, \ldots, 10$ of this surface are called the *quasi-momenta* and are parametrized by a spectral parameter $x \in \mathbb{C}$. Pairs of these sheets can be connected by square root cuts $\mathcal{C}_{ij}$. When going through the cut the quasi-momenta get shifted by an integer multiple of 2π

$$q_i(x + i\epsilon) - q_j(x - i\epsilon) = 2\pi n_{ij}, \tag{1}$$

where q_i and q_j are evaluated on opposing side of the cut, and $n_{ij} \in \mathbb{Z}$ are called *mode numbers.*

The charges of the string state corresponding to a specific curve is given by the inversion symmetry and the curve's asymptotic behavior at large x. Some important properties of the algebraic curve are summarized in App. B.

2.2 Ansatz for the algebraic curve in the SU(4) sector

Our aim is to find quasi-momenta $q_1(x), \ldots, q_{10}(x)$ with the correct poles and symmetries, and having the right large x asymptotics. In this paper we will treat the $\mathrm{SU}(2)\times\mathrm{SU}(2) \subset \mathrm{SU}(4)$ sector and use the ansatz [24]

$$q_1(x) = -q_{10}(x) = \alpha\frac{x}{x^2-1}, \tag{2}$$

$$q_2(x) = -q_9(x) = \alpha\frac{x}{x^2-1}, \tag{3}$$

$$\begin{aligned} q_3(x) = -q_8(x) = \alpha\frac{x}{x^2-1} + G_r(x) + G_r\left(\frac{1}{x}\right) - G_v\left(\frac{1}{x}\right) - G_u\left(\frac{1}{x}\right) \\ - G_r(0) + G_v(0) + G_u(0), \end{aligned} \tag{4}$$

$$q_4(x) = -q_7(x) = \alpha\frac{x}{x^2-1} + G_v(x) + G_u(x) - G_r(x) - G_r\left(\frac{1}{x}\right) + G_r(0), \tag{5}$$

$$q_5(x) = -q_6(x) = -G_v(x) + G_u(x) - G_v\left(\frac{1}{x}\right) + G_u\left(\frac{1}{x}\right) + G_v(0) - G_u(0). \tag{6}$$

The subscripts of the resolvents G_v, G_u and G_r correspond to the excitation numbers of App. A, and indicate which Dynkin labels of SU(4) are excited by a cut in the resolvent.

2.3 SU(2) giant magnon

As a simple check of the ansatz (2)-(6) we will derive the dispersion relation of a single SU(2) giant magnon. The resolvents then take the form

$$G_v(x) = \frac{1}{i}\log\frac{x-X^+}{x-X^-}, \qquad G_u(x) = G_r(x) = 0. \tag{7}$$

In order to obtain conserved charges of the magnon we have to consider the large x behavior of the quasi-momenta, and compare it with the expected limits from App. B[3]

$$q_1(x) = q_2(x) = \frac{\alpha x}{x^2} + \cdots = \frac{E \pm S}{2gx} + \cdots, \tag{8}$$

$$q_4(x) + q_3(x) = -\frac{i}{x}\left(X^+ - X^- - \frac{1}{X^+} + \frac{1}{X^-} + 2i\alpha\right) + \cdots = -\frac{J}{2gx} + \cdots \tag{9}$$

$$q_5(x) = q_4(x) - q_3(x) = -\frac{i}{x}\left(X^+ - X^- + \frac{1}{X^+} - \frac{1}{X^-}\right) + \cdots = -\frac{Q}{2gx} + \cdots. \tag{10}$$

and we can find from (8) that $E = 2g\alpha$ and $S = 0$. To check the inversion symmetry we calculate[4]

$$\pi m = q_3(1/x) + q_4(x) = -i\log\frac{X^+}{X^-} \equiv p. \tag{11}$$

Solving (10) together with the momentum equation (11) for $X^\pm$ we get

$$X^\pm = \frac{\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + 16g^2\sin\frac{p}{2}}}{4g\sin\frac{p}{2}}\, e^{\pm i\frac{p}{2}}. \tag{12}$$

Plugging this into (9) gives the dispersion relation

$$E - \frac{J}{2} = \sqrt{\frac{Q^2}{4} + 16g^2\sin^2\frac{p}{2}} = \sqrt{\frac{Q^2}{4} + 2\lambda\sin^2\frac{p}{2}}. \tag{13}$$

This dispersion relation for the SU(2) magnon is the same as the "small" giant magnon dispersion relation considered by Gaiotto *et al.* [31] and by Shenderovich [36].

2.3.1 Finite size corrections to SU(2) giant magnon

Let us continue by computing the finite size correction to a single magnon in the SU(2) sector. Inspired by [48] we use the resolvents[5]

$$G_v(x) = G(x) = -2i\log\frac{\sqrt{x-X^+} + \sqrt{x-Y^+}}{\sqrt{x-X^-} + \sqrt{x-Y^-}}, \qquad G_u(x) = G_r(x) = 0. \tag{14}$$

[3] The coupling g is related to the 't Hooft coupling λ by

$$\lambda = 8g^2.$$

[4] When considering a single giant magnon we can relax the level matching condition so that $p \notin \pi\mathbb{Z}$.

[5] These resolvents was used in [57] to calculate the finite size corrections to the giant magnon dispersion relation in $\mathcal{N} = 4$ SYM.

The function $G(x)$ has a log cut between the points X^+ and X^- and two square root cuts connecting $X^\pm$ and $Y^\pm$. In the limit $Y^\pm \to X^\pm$, the resolvent $G(x) \to -i\log\frac{x-X^+}{x-X^-}$, which gives the previous single magnon solution.

The momentum of the magnon can be found from the inversion symmetry

$$p = q_3(1/x) + q_4(x) = -2i\log\frac{\sqrt{X^+}+\sqrt{Y^+}}{\sqrt{X^-}+\sqrt{Y^-}}. \tag{15}$$

and the conserved charges from the large x asymptotics

$$\frac{J}{2g} \approx \frac{E}{g} + \frac{i}{2}\left(X^+ - X^- + Y^+ - Y^- - \frac{2}{\sqrt{X^+Y^+}} + \frac{2}{\sqrt{X^-Y^-}}\right), \tag{16}$$

$$\frac{Q}{2g} \approx -\frac{i}{2}\left(X^+ - X^- + Y^+ - Y^- + \frac{2}{\sqrt{X^+Y^+}} - \frac{2}{\sqrt{X^-Y^-}}\right). \tag{17}$$

To solve the equations (16) and (17) we introduce

$$i\delta e^{i\phi} = Y^+ - X^+, \tag{18}$$

and solve the equations perturbatively in δ (for $g \gg 1$). The result is

$$E - \frac{J}{2} = 4g\sin\frac{p}{2} - g\frac{\delta^2}{4}\sin\frac{p}{2}\cos(p-2\phi). \tag{19}$$

In order to calculate δ and ϕ we need to use the condition that the sheets q_4 and q_5 are connected by square root cuts. This reads

$$q_4(x+i\epsilon) - q_5(x-i\epsilon) = 2\pi n, \quad x \in \mathcal{C}, \tag{20}$$

where $\mathcal{C}$ is one of the cuts. Focusing on the upper cut we get the condition

$$2\pi n = \frac{E}{2g}\frac{x}{x^2-1} + G(x+i\epsilon) + G(x-i\epsilon) + G(1/x) - G(0). \tag{21}$$

The first part of the right hand side is the same as in the $\mathcal{N}=4$ case, so we can incorporate the result from that case, which is

$$G(x+i\epsilon) + G(x-i\epsilon) = -2i\log\frac{Y^+-X^+}{x-X^-} + 4i\log\left(1+\sqrt{\frac{x-Y^-}{x-X^-}}\right). \tag{22}$$

We are interested in the leading order behavior as $Y^\pm \to X^\pm$ in the formula (22). Hence we can evaluate it at $x = X^+$. We then get

$$\begin{aligned}\frac{E}{2g}\frac{x}{x^2-1} + G(x+i\epsilon) + G(x-i\epsilon) &\approx \frac{E}{2g}\frac{X^+}{X^{+2}-1} + G(X^+ + i\epsilon) + G(X^+ - i\epsilon) + \mathcal{O}(\delta)\\ &\approx \frac{E}{2g}\frac{X^+}{X^{+2}-1} - 2i\log\frac{ie^{i\phi}\delta}{4(X^+-X^-)} + \mathcal{O}(\delta)\\ &\approx -i\frac{E}{4g\sin\frac{p}{2}} - 2i\log\frac{e^{i\phi}\delta}{8\sin\frac{p}{2}} + \mathcal{O}(\delta).\end{aligned}$$

The last two terms in (21) do not appear in the $\mathcal{N}=4$ case and need to be treated a bit more carefully. They are given by

$$\begin{aligned} G(1/X^+) - G(0) &= -i\log\frac{\frac{1}{X^+}-X^+}{\frac{1}{X^+}-X^-} + i\log\frac{X^+}{X^-} + \mathcal{O}(\delta) \\ &= -i\log\left(\cos\frac{p}{2} + i\sin\frac{p}{2}\frac{\sqrt{\frac{Q^2}{4}+16g^2\sin^2\frac{p}{2}}}{\frac{Q}{2}}\right) - \frac{p}{2} + \mathcal{O}(\delta) \\ &\approx -i\log\frac{8ig\sin^2\frac{p}{2}}{Q} - \frac{p}{2} + \mathcal{O}(\delta). \end{aligned}$$

Collecting the terms we get the condition

$$2\pi n = -i\frac{E}{4g\sin\frac{p}{2}} - 2i\log\frac{e^{i\phi}\delta}{8\sin\frac{p}{2}} - i\log\frac{8ig\sin^2\frac{p}{2}}{Q} - \frac{p}{2} + \mathcal{O}(\delta), \tag{23}$$

which gives

$$\delta = \sqrt{\frac{8Q}{g}e^{-\frac{E}{8g\sin\frac{p}{2}}}}, \qquad \phi = \frac{p}{4} + n\pi \pm \frac{\pi}{4}, \tag{24}$$

where the sign of the last term depends on how we chose the branch of $\frac{1}{2}\log i$. The finite size dispersion relation is now given by

$$E - \frac{J}{=}4g\sin\frac{p}{2} \pm 2Q\sin\frac{p}{2}\sin\left(\frac{p}{2} - 2\pi n\right)e^{-\frac{E}{4g\sin\frac{p}{2}}}. \tag{25}$$

The form of this correction is very different from the one in the $\mathcal{N}=4$ case, since the leading order correction is suppressed by a factor $1/g$ in addition to the exponential suppression. Moreover the $\mathcal{N}=4$ corrections are independent of the charge Q for $Q \ll g$. In the present case, the leading corrections vanish if we let $Q \to 0$.

To identify more easily the correction we can consider a physical state consisting of M magnons with momentum p and charge Q. This is described by shifting the resolvent $G(x) \to M \cdot G(x)$. The correction is now given by

$$E - \frac{J}{=}4Mg\sin\frac{p}{2}\left[1 \pm \frac{Q}{2g}\sin\left(\frac{p}{2} - \frac{2\pi n}{M}\right)e^{-\frac{E/M}{4g\sin\frac{p}{2}}}\right]. \tag{26}$$

For a physical configuration $p = \frac{\pi m}{M}$ for some integer m. For a fundamental magnon $(Q=1)$ we get

$$\delta\mathcal{E} = 2\sin^2\frac{p}{2}e^{-\frac{E}{4g\sin\frac{p}{2}}}, \qquad n = 0 \tag{27}$$

$$\delta\mathcal{E} = 0, \qquad n = \frac{p}{4\pi}. \tag{28}$$

2.4 SU(2) × SU(2) giant magnon

We now want to consider giant magnons in the SU(2) × SU(2) sector. The simplest configuration consists of one fundamental magnon in each SU(2) sector, with equal momenta p. For this case can use the ansatz (2)–(6) with

$$G_u(x) = G_v(x) = G(x) = -2i \log \frac{\sqrt{x - X^+} + \sqrt{x - Y^+}}{\sqrt{x - X^-} + \sqrt{x - Y^-}} \tag{29}$$

and $G_r(x) = 0$. Following the same procedure as in the SU(2) case this gives

$$E - J = 8g \sin \frac{p}{2} - g \frac{\delta^2}{2} \sin \frac{p}{2} \cos(p - 2\phi). \tag{30}$$

Again we need to consider the condition that the quasi-momenta should have square root cuts. The two cuts are at the same position, but connect different sheets. In order to write down the condition we imagine separating them slightly, so that we can consider two points on opposite sides of one of the cuts, but on the same side of the other. Our condition is then

$$2\pi n = q_4(x + i\epsilon) - q_5(x - i\epsilon) = \frac{E}{2g} \frac{x}{x^2 - 1} + G(x + i\epsilon) + G(x - i\epsilon). \tag{31}$$

Note that the terms of the kind $G(1/x) - G(0)$ exactly cancel between the two magnons. Equation (31) is identical to the corresponding equation in $\mathcal{N} = 4$, and the solution is

$$\delta = 8 \sin \frac{p}{2} e^{-\frac{E}{8g \sin \frac{p}{2}}}, \qquad \phi = -\pi - \pi n. \tag{32}$$

Thus the finite size dispersion relation for this configuration is

$$\mathcal{E} = E - J = 8g \sin \frac{p}{2} \left[1 - 4 \sin^2 \frac{p}{2} \cos(p - 2\pi n) e^{-\frac{E}{4g \sin \frac{p}{2}}} \right]. \tag{33}$$

Again a simple generalization to M equal magnons in each sector leads to two natural choices for n:

$$\delta\mathcal{E} = -32g \sin^3 \frac{p}{2} \cos p \, e^{-\frac{E}{4g \sin \frac{p}{2}}}, \qquad n = 0, \tag{34}$$

$$\delta\mathcal{E} = -32g \sin^3 \frac{p}{2} e^{-\frac{E}{4g \sin \frac{p}{2}}}, \qquad n = \frac{p}{2\pi}. \tag{35}$$

2.4.1 General multi-magnon states

Using the algebraic curve we can also calculate the finite size corrections to a general multi-magnon state in the SU(2) × SU(2) sector. Hence we consider a state consisting of M magnons in the $\mathrm{SU}(2)_v$ sector and $\hat{M}$ magnons in the $\mathrm{SU}(2)_u$ sector, having momenta p_i and $\hat{p}_i$ respectively.

At infinite J, the dispersion relation will be given by

$$\mathcal{E}_\infty = \sum^{M} \mathcal{E}_i + \sum^{\hat{M}} \hat{\mathcal{E}}_i, \qquad \mathcal{E}_i = 4g \sin \frac{p_i}{2}, \quad \hat{\mathcal{E}}_i = 4g \sin \frac{\hat{p}_i}{2}. \tag{36}$$

At finite J this will get corrections, and we will write

$$\mathcal{E} = \sum_{i=1}^{M}\Big(\mathcal{E}_i + \delta\mathcal{E}_i\Big) + \sum_{i=1}^{\hat{M}}\Big(\hat{\mathcal{E}}_i + \delta\hat{\mathcal{E}}_i\Big). \tag{37}$$

As an ansatz for the algebraic curve, we use a generalization of the previous one with

$$G_v(x) = \sum_{i=1}^{M} G_i(x) = \sum_{i=1}^{M}\left(-2i\log\frac{\sqrt{x - X_i^+} + \sqrt{x - Y_i^+}}{\sqrt{x - X_i^-} + \sqrt{x - Y_i^-}}\right), \tag{38}$$

$$G_u(x) = \sum_{i=1}^{M} \hat{G}_i(x) = \sum_{i=1}^{\hat{M}}\left(-2i\log\frac{\sqrt{x - \hat{X}_i^+} + \sqrt{x - \hat{Y}_i^+}}{\sqrt{x - \hat{X}_i^-} + \sqrt{x - \hat{Y}_i^-}}\right). \tag{39}$$

For definiteness let us consider the first magnon in $\mathrm{SU}(2)_v$. Following the previous procedure we get

$$\delta\mathcal{E}_1 = -g\frac{\delta^2}{4}\sin\frac{p_1}{2}\cos(p_1 - 2\phi). \tag{40}$$

Again we calculate δ and ϕ by requiring that

$$q_4(x + i\epsilon) - q_5(x - i\epsilon) = 2\pi n. \tag{41}$$

Writing this out we get for x in $\mathcal{C}_1^+$, the cut connecting the branch points X_1^+ and Y_1^+,

$$\begin{aligned}2\pi n = \frac{E}{2g}\frac{x}{x^2-1} &+ G_1(x+i\epsilon) + G_1(x-i\epsilon) + G_1(1/x) - G_1(0)\\ &+ \sum_{i=2}^{M}\Big(G_i(1/x) - G_i(0)\Big) - \sum_{i=1}^{\hat{M}}\Big(\hat{G}_i(1/x) - \hat{G}_i(0)\Big).\end{aligned} \tag{42}$$

The first row of this equation is identical to the one in the one-magnon case. The second row induces interactions between the magnons. From our previous results we have

$$\begin{aligned}\frac{E}{2g}\frac{x}{x^2-1} &+ G_1(x+i\epsilon) + G_1(x-i\epsilon) + G_1(1/x) - G_1(0) \approx\\ &- i\frac{E}{4g\sin\frac{p_1}{2}} - 2i\log\frac{e^{i\phi}\delta}{8\sin\frac{p_1}{2}} - i\log\frac{8ig\sin^2\frac{p_1}{2}}{Q_1} - \frac{p_1}{2} + \mathcal{O}(\delta).\end{aligned} \tag{43}$$

Moreover

$$\begin{aligned}G_i\left(\frac{1}{x}\right) - G_i(0) &\approx G_i\left(\frac{1}{X_1^+}\right) - G_i(0)\\ &\approx -i\log\frac{\frac{1}{X_1^+} - X_i^+}{\frac{1}{X_1^+} - X_i^-} + i\log\frac{X_i^+}{X_i^-}\\ &\approx -i\log\frac{\sin\frac{p_1+p_i}{4}}{\sin\frac{p_1-p_i}{4}} - \frac{p_i}{2},\end{aligned}$$

and similarly for $\hat{G}_i$. Thus

$$\sum_{i=2}^{M}\Big(G_i(1/x)-G_i(0)\Big)-\sum_{i=1}^{\hat{M}}\Big(\hat{G}_i(1/x)-\hat{G}_i(0)\Big)\approx$$
$$-\,i\log\left(\prod_{i=2}^{M}\frac{\sin\frac{p_1+p_i}{4}}{\sin\frac{p_1-p_i}{4}}\right)+i\log\left(\prod_{i=1}^{\hat{M}}\frac{\sin\frac{p_1+\hat{p}_i}{4}}{\sin\frac{p_1-\hat{p}_i}{4}}\right)-\sum_{i=2}^{M}\frac{p_i}{2}+\sum_{i=1}^{\hat{M}}\frac{\hat{p}_i}{2}. \quad (44)$$

Collecting these results we get

$$\delta\mathcal{E}_1=2Q_1\sin\frac{p_1}{2}\prod_{i=2}^{M}\frac{\sin^2\frac{p_1-p_i}{4}}{\sin^2\frac{p_1+p_i}{4}}\prod_{i=1}^{\hat{M}}\frac{\sin^2\frac{p_1+\hat{p}_i}{4}}{\sin^2\frac{p_1-\hat{p}_i}{4}}$$
$$\times\sin\left(p_1-\sum_{i=1}^{M}\frac{p_i}{2}+\sum_{i=1}^{\hat{M}}\frac{\hat{p}_i}{2}+2\pi n\right)\,e^{-\frac{E}{4g\sin\frac{p_1}{2}}}. \quad (45)$$

As in $\mathcal{N}=4$, the contribution from the magnon interactions is related to the magnon S-matrix [48]. Note that magnons in the same sector contribute with a different sign than magnons in the opposite sector.

3 Finite size corrections from the Lüscher μ-term

The second approach to the finite size effects is based on the so called Lüscher formulae obtained for the first time by Lüscher [58] for a relativistic field theory on a cylinder and derived in [39] for general dispersion relations. We will focus only on the μ-term which is given by [46]

$$\delta\mathcal{E}_a^{\mu}=-i\left(1-\frac{\mathcal{E}'(p)}{\mathcal{E}'(\tilde{q}_*)}\right)\,e^{iq_*}\cdot\operatorname*{res}_{q=\tilde{q}}\sum_b S_{ba}^{ba}(q_*,p). \quad (46)$$

Many of the following results can be easy obtained from the $\mathrm{AdS}_5\times S^5$ case.

3.1 SU(2) giant magnon

We start from the computations for an SU(2) giant magnon. The dispersion relation of a fundamental giant magnon in $\mathrm{AdS}_4\times\mathbb{CP}^3$ is given by

$$\mathcal{E}_4=E-\frac{J}{2}=\sqrt{\frac{1}{4}+16g^2\sin^2\frac{p}{2}}, \quad (47)$$

while the corresponding relation for the $\mathrm{AdS}_5\times S^5$ case is

$$\mathcal{E}_5=E-J=\sqrt{1+16g^2\sin^2\frac{p}{2}}. \quad (48)$$

Note that $2\mathcal{E}_4$ equals $\mathcal{E}_5$ if we shift $g \to 2g$ and $E \to 2E$ in $\mathcal{E}_5$. Hence we can import kinematical results from $\mathcal{N}=4$ to $\mathcal{N}=6$, provided we make this shift of the energy and the coupling.

The matrix part cannot be obtained so easily from the $\mathrm{AdS}_5 \times S^5$ case so we have give it some more attention. As described in [25], there are two types of fundamental excitations in $\mathcal{N}=6$ superconformal Chern-Simons theory. We will refer to these as excitations of type A and B. Correspondingly the S-matrix can be divided into two parts – the matrices S^{AA} and S^{BB} describing scattering of particles of the same type, and the matrices S^{AB} and S^{BA} describing scattering of particles of different types. We write these S-matrices as

$$S^{AA}(p_1,p_2) = S^{BB}(p_1,p_2) = S_0(p_1,p_2)\hat{S}(p_1,p_2), \tag{49}$$
$$S^{AB}(p_1,p_2) = S^{BA}(p_1,p_2) = \tilde{S}_0(p_1,p_2)\hat{S}(p_1,p_2), \tag{50}$$

where $\hat{S}$ is the SU(2|2)-invariant S-matrix of [59] with g appropriately shifted as noted above. The scalar factors S_0 and $\tilde{S}_0$ are given by

$$S_0(p_1,p_2) = \frac{1-\frac{1}{x_1^+ x_2^-}}{1-\frac{1}{x_1^- x_2^+}}\sigma(p_1,p_2), \tag{51}$$
$$\tilde{S}_0(p_1,p_2) = \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\sigma(p_1,p_2), \tag{52}$$

where $\sigma(p_1,p_2)$ is the BES dressing factor [60].

The relevant S-matrix coefficients are

$$a_1 = \frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2} \tag{53}$$
$$a_2 = \frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)}{x_1^+ x_2^+ - x_1^- x_2^-}\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2} \tag{54}$$
$$a_6 = \frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{\eta_2}{\tilde{\eta}_2}. \tag{55}$$

The phase factors η depend on the choice of basis. In the *string frame*

$$\frac{\eta_1}{\tilde{\eta}_1} = \sqrt{\frac{x_2^+}{x_2^-}}, \qquad \frac{\eta_2}{\tilde{\eta}_2} = \sqrt{\frac{x_1^-}{x_1^+}}, \tag{56}$$

while in the *spin chain frame*

$$\frac{\eta_1}{\tilde{\eta}_1} = \frac{\eta_2}{\tilde{\eta}_2} = 1. \tag{57}$$

We will consider a single fundamental magnon of A-type. In order to calculate the Lüscher μ-term, we need to know the poles of the S-matrix. Using the above expressions for the SU(2) sector we see that $S^{BA}(p_1,p_2)$ has no poles while $S^{AA}(p_1,p_2)$ has a physical pole at $x_1^- = x_2^+$. The position of this pole is the same as for a single SU(2) magnon in $\mathcal{N}=4$. Since the pole positions agree, we can directly import the result for the kinematical part from [46]. Thus

$$\delta\mathcal{E}_a^\mu = -\frac{i}{2}\sin^2\frac{p}{2}e^{-\frac{J}{8g\sin\frac{p}{2}}}\cdot\underset{q=\tilde{q}}{\mathrm{res}}\sum_b S_{ba}^{ba}(q_*,p). \tag{58}$$

Following [46] we can express the S-matrix in terms of a_i

$$\sum_b S_{ab}^{ab}(q_*,p) = S_0(q_*,p)(2a_1+a_2+2a_6). \tag{59}$$

and using the formulae for a_i obtain the result which depends only on the frame we choose

$$\underset{q\to\tilde{q}}{\mathrm{res}}\sum_b S_{ab}^{ab}(q_*,p) = \frac{1}{x_1^{-\prime}}\cdot\underset{x_1^-\to x_2^+}{\mathrm{res}}\sum_b S_{ab}^{ab}(q_*,p) \tag{60}$$

$$= \frac{ie^{-i\frac{p}{2}}}{\sin^2\frac{p}{2}}\cdot\underset{x_1^-\to x_2^+}{\mathrm{res}}\sum_b S_{ab}^{ab}(q_*,p) \tag{61}$$

$$= \frac{i}{g\sin^3\frac{p}{2}}\cdot\frac{\eta_1}{\tilde{\eta}_1}\frac{\eta_2}{\tilde{\eta}_2}\cdot\sigma(x_1,x_2). \tag{62}$$

Now we can plug it into the formula for μ-term

$$\delta\mathcal{E}_a^\mu = \frac{e^{-\frac{J}{4g\sin\frac{p}{2}}}}{2g\sin\frac{p}{2}}\cdot\frac{\eta_1}{\tilde{\eta}_1}\frac{\eta_2}{\tilde{\eta}_2}\cdot\sigma(x_1,x_2). \tag{63}$$

The value of the dressing factor at the pole is given by the same expression as in $\mathcal{N}=4$, namely [46]

$$\sigma^2(x_1,x_2) = -\frac{16g^2}{e^2}e^{-ip}\sin^4\frac{p}{2}. \tag{64}$$

Putting things together the μ-term is

$$\delta\mathcal{E}_a^\mu = \frac{2i}{e}\sin\frac{p}{2}e^{-\frac{J}{8g\sin\frac{p}{2}}}, \qquad \text{string frame,} \tag{65}$$

$$\delta\mathcal{E}_a^\mu = \frac{2i}{e}\sin\frac{p}{2}e^{-\frac{J}{8g\sin\frac{p}{2}}}e^{-i\frac{p}{2}}, \qquad \text{spin chain frame.} \tag{66}$$

The correction to the dispersion relation should be real. Taking the real part of the above expressions we get

$$\delta\mathcal{E} = 0, \qquad \text{string frame,} \tag{67}$$

$$\delta\mathcal{E} = \frac{2}{e}\sin^2\frac{p}{2}e^{-\frac{J}{8g\sin\frac{p}{2}}} = 2\sin^2\frac{p}{2}e^{-\frac{E}{4g\sin\frac{p}{2}}}, \qquad \text{spin chain frame.} \tag{68}$$

We can now compare this result to the result of the algebraic curve calculation. If we consider a fundamental magnon with $Q=1$ and let $n=0$ in (25) we get exactly the above result from the spin chain frame. Choosing $n=p/4\pi$ gives a vanishing correction, like in the string frame.

3.2 SU(2) × SU(2) giant magnon

In order to calculate the corrections to a multi-magnon state we need the generalized Lüscher formula of Hatsuda and Suzuki [47][6]. The two-magnon μ-term is given by

$$\delta\mathcal{E}^{\mu}_{a_1 a_2} = 2\sum_b (-1)^{F_b}\left[1-\frac{\mathcal{E}'_{a_1}(p_1)}{\mathcal{E}'_b(q_1^*)}\right] e^{-iq_1^* J} \operatorname*{res}_{q=\tilde{q}_1^*} S^{ba_1}_{ba_1}(q^1,p_1) S^{ba_2}_{ba_2}(q_1^*,p_2). \tag{69}$$

Since the two magnons are in different SU(2) sectors, one of the S-matrices will be of the type S^{AA} or S^{BB}, while the other will be of the type S^{AB} or S^{BA}. Hence the full S-matrix factor will be of the form

$$S_0(q,p)\tilde{S}_0(q,p)\hat{S}^{1b}_{1b}(q,p)\hat{S}^{1b}_{1b}(q,p). \tag{70}$$

But this is the exact same structure as for the $SU(2|2)^2$ S-matrix of $\mathcal{N}=4$. Moreover, the full μ-term now has the form of the one magnon correction in $\mathcal{N}=4$. Thus we can just use the result of Janik and Łukowski [46] and write

$$\delta\mathcal{E} = \mathrm{Re}\left[-32g\sin^2\frac{p}{2}e^{-\frac{E}{4g\sin\frac{p}{2}}}\left(\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\right)^2\right]. \tag{71}$$

Again there are two choices for the phase factors η:

$$\delta\mathcal{E} = -32g\sin^3\frac{p}{2}e^{-\frac{E}{4g\sin\frac{p}{2}}} \qquad \text{string frame,} \tag{72}$$

$$\delta\mathcal{E} = -32g\sin^3\frac{p}{2}\cos(p)e^{-\frac{E}{4g\sin\frac{p}{2}}} \qquad \text{spin chain frame.} \tag{73}$$

4 Comparing the results

The calculation of the finite size corrections to the two magnon configuration in SU(2) × SU(2) which we considered, closely follows the calculation of finite size corrections for a single magnon in $AdS_5 \times S^5$. In the string frame our final result was

$$E = 8g\sin\frac{p}{2}\left(1-4\sin^2\frac{p}{2}e^{-\frac{E}{4g\sin\frac{p}{2}}}\right) \tag{74}$$

$$= 2\sqrt{2\lambda}\sin\frac{p}{2}\left(1-\frac{4}{e^2}\sin^2\frac{p}{2}e^{-\frac{J}{\sqrt{2\lambda}\sin\frac{p}{2}}}\right) \tag{75}$$

As in that case we find perfect agreement between the results of the finite gap and Lüscher calculations. Similar to the SU(2) magnon there is a correspondence between the choice of frame for the S-matrix when calculating the Lüscher term, and the choice of branch, or mode number, in the finite gap system.

[6] Essentially the same formula was independently given by Bajnok and Janik [43].

5 Conclusions

In this paper we studied the finite size corrections for giant magnon states in the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ sector using the algebraic curve as well as the Lüscher μ-term. For the case of one excitation in each SU(2), with both excitations carrying the same momenta, the resulting corrections perfectly match those of previous calculations [35, 51, 52]. It is encouraging that both the algebraic curve and the Lüscher term give the same result as a direct string theory calculation.

The result for a single SU(2) magnon is a bit harder to interpret, since the result of the Lüscher term is not real. In itself this could be a sign that some contributions, such as those of the bound states, are missing. However, the real part of the result perfectly matches the result from the algebraic curve. Moreover the choice of the string frame versus spin-chain frame in the SU(2|2) S-matrix corresponds to different choices of the mode number of the curve.[7] The agreement between the two calculations give a good consistency check between the algebraic curve [23] and the S-matrix proposed in [25].

The generic correction is proportional to the R-charge Q, and not to g as in $\mathcal{N} = 4$. Hence the classical correction vanishes for fundamental magnons. From the algebraic curve perspective, it seems like setting $Q = 0$ forces the finite size magnon curve back to a curve describing an infinite J magnon. An explicit sigma model construction of a single finite size SU(2) magnon might lead to an interpretation of this result.

The exceptional case is when we have two magnons with equal momenta. The corrections are then enhanced to become finite. In both the Lüscher and finite gap calculations this can be traced back to the appearance of extra singularities.

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A Notation

The SU(4) Dynkin labels $[p_1, q, p_2]$ are related to the operator length L and the excitation numbers M_u, M_v and M_r by

$$[p_1, q, p_2] = [L - 2M_u + M_r, M_u + M_v - 2M_r, L - 2M_v + M_r]. \tag{76}$$

[7] Also for $\mathcal{N} = 4$ the choice of basis for the S-matrix in the Lüscher term corresponds to a choice of mode numbers for the algebraic curve. However, the Lüscher term is real in the string frame, so only this case has been generally considered.

We assign the SO(6) $\cong$ SU(4) R-charges J_1, J_2 and J_3 as

$$J_1 = q + \frac{p_2 + p_1}{2} = L - M_r, \tag{77}$$

$$J_2 = \frac{p_2 + p_1}{2} = L + M_r - M_u - M_v, \tag{78}$$

$$J_3 = \frac{p_2 - p_1}{2} = M_u - M_v, \tag{79}$$

We also introduce the charges

$$J = J_1 + J_2 = 2L - M_u - M_v \quad \text{and} \quad Q = J_1 - J_2 = M_u + M_v - 2M_r. \tag{80}$$

B Properties of algebraic curve

This appendix summarize some properties of the quasi-momenta of the algebraic curve for $\mathcal{N} = 6$ superconformal Chern-Simons.

- dependence of quasi-momenta

$$\begin{pmatrix} q_1(x) \\ q_2(x) \\ q_3(x) \\ q_4(x) \\ q_5(x) \end{pmatrix} = - \begin{pmatrix} q_{10}(x) \\ q_9(x) \\ q_8(x) \\ q_7(x) \\ q_6(x) \end{pmatrix} \tag{81}$$

- condition for cuts

$$q_i(x + i\epsilon) - q_j(x - i\epsilon) = 2\pi n_{ij} \tag{82}$$

- synchronization of poles at $x = \pm 1$

$$\begin{pmatrix} q_1(x) \\ q_2(x) \\ q_3(x) \\ q_4(x) \\ q_5(x) \end{pmatrix} = - \begin{pmatrix} q_{10}(x) \\ q_9(x) \\ q_8(x) \\ q_7(x) \\ q_6(x) \end{pmatrix} = \frac{1}{2}\frac{1}{x \mp 1} \begin{pmatrix} \alpha_\pm \\ \alpha_\pm \\ \alpha_\pm \\ \alpha_\pm \\ 0 \end{pmatrix} + \cdots \tag{83}$$

- inversion symmetry ($m \in \mathbb{Z}$)

$$\begin{pmatrix} q_1(1/x) \\ q_2(1/x) \\ q_3(1/x) \\ q_4(1/x) \\ q_5(1/x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \pi m \\ \pi m \\ 0 \end{pmatrix} + \begin{pmatrix} -q_2(x) \\ -q_1(x) \\ -q_4(x) \\ -q_3(x) \\ +q_5(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \pi m \\ \pi m \\ 0 \end{pmatrix} + \begin{pmatrix} +q_9(x) \\ +q_{10}(x) \\ +q_7(x) \\ +q_8(x) \\ -q_6(x) \end{pmatrix} \tag{84}$$

- asymptotic behavior at $x \to \infty$

$$\begin{pmatrix} q_1(x) \\ q_2(x) \\ q_3(x) \\ q_4(x) \\ q_5(x) \end{pmatrix} = \frac{1}{2gx} \begin{pmatrix} E + S \\ E - S \\ L - M_r \\ L + M_r - M_u - M_v \\ M_v - M_u \end{pmatrix} = \frac{1}{2gx} \begin{pmatrix} E + S \\ E - S \\ J_1 \\ J_2 \\ -J_3 \end{pmatrix} \tag{85}$$

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